How to Fix College Football with Linear Algebra

A final project for MATH 315

Summer 2025

The Problem

College football poll era (1936-1991)

- Conferences based on geography
- Conference champions played in pre-determined bowls
  - Example: Rose Bowl (Big 10 vs Pac 10)
- Human polls determined who was the "national champion"

College football poll era (1936-1991)

- Pros
  - Geography mattered
  - Small conferences had clear champions
  - History and tradition
- Cons
  - Stagnant rankings
  - Inconsistent matchups
    - 1963 Rose Bowl: #1 USC vs #2 Wisconsin
    - 2013 Rose Bowl: #8 Stanford vs Wisconsin (unranked, 8-5 record) in a game that featured a total of 3 second-half points
  - Split national championships
    - Back-to-back split NCs in 1990 and 1991 pushed the sport to its breaking point

College football BCS era (1992-2013)

- IDEA: Conferences agree to let the two best teams play in a bowl for the national championship, regardless of conference affiliation
  - Essentially switches the order of the bowls and human polls
- Problems remained
  - Tradition: Big 10 and Pac 10 refused to participate at first, leading to another split NC in 1997
  - Access: Smaller schools had no realistic chance of playing for the championship (notably Utah, Boise St, and TCU)
  - Controversy: 2004 had five undefeated teams, and only two could play for the title (believe it or not, the SEC team was left out!)
    - These five schools finished the year ranked in the exact same order as in the preseason poll

College football playoff era (2014-present)

- Started with 4 teams, expanded to 12 for 2024 season
- SEC teams have dominated, winning 6 of first 10 championships
- The landscape continues to shift with NIL and conference realignment
    - Big conferences create unbalanced schedules and controversial champions

The elephant in the room: unbalanced schedules

- 2024 Big Ten Schedules for Indiana (8-1, qualified for College Football Playoff) and Minnesota (5-4, qualified for Duke's Mayo Bowl)
- They did not play against each other
- Only three of nine opponents in common
  - Both beat Maryland at home
  - Both beat UCLA on the road
  - Indiana beat Michigan by 5 at home, Minnesota lost to Michigan by 3 on the road

The elephant in the room: unbalanced schedules

INDIANA-HOME	
Oregon	9-0
Penn State	8-1
Indiana	8-1
Ohio State	7-2
Illinois	6-3
Iowa	6-3
Michigan	**5-4**
Minnesota	5-4
Rutgers	4-5
Washington	**4-5**
USC	4-5
UCLA	3-6
Nebraska	**3-6**
Michigan State	3-6
Wisconsin	3-6
Northwestern	2-7
Maryland	**1-8**
Purdue	**0-9**

MINNESOTA-HOME	
Oregon	9-0
Penn State	**8-1**
Indiana	8-1
Ohio State	7-2
Illinois	6-3
Iowa	**6-3**
Michigan	5-4
Minnesota	5-4
Rutgers	4-5
Washington	4-5
USC	**4-5**
UCLA	3-6
Nebraska	3-6
Michigan State	3-6
Wisconsin	3-6
Northwestern	2-7
Maryland	**1-8**
Purdue	0-9

The elephant in the room: unbalanced schedules

INDIANA-AWAY	
Oregon	9-0
Penn State	8-1
Indiana	8-1
Ohio State	**7-2**
Illinois	6-3
Iowa	6-3
Michigan	5-4
Minnesota	5-4
Rutgers	4-5
Washington	4-5
USC	4-5
UCLA	**3-6**
Nebraska	3-6
Michigan State	**3-6**
Wisconsin	3-6
Northwestern	**2-7**
Maryland	1-8
Purdue	0-9

MINNESOTA-AWAY	
Oregon	9-0
Penn State	8-1
Indiana	8-1
Ohio State	7-2
Illinois	**6-3**
Iowa	6-3
Michigan	**5-4**
Minnesota	5-4
Rutgers	**4-5**
Washington	4-5
USC	4-5
UCLA	**3-6**
Nebraska	3-6
Michigan State	3-6
Wisconsin	**3-6**
Northwestern	2-7
Maryland	1-8
Purdue	0-9

The elephant in the room: unbalanced schedules

INDIANA	
Oregon	9-0
Penn State	8-1
Indiana	8-1
Ohio State	**7-2**
Illinois	6-3
Iowa	6-3
Michigan	**5-4**
Minnesota	5-4
Rutgers	4-5
Washington	**4-5**
USC	4-5
UCLA	**3-6**
Nebraska	**3-6**
Michigan State	**3-6**
Wisconsin	3-6
Northwestern	**2-7**
Maryland	**1-8**
Purdue	**0-9**

MINNESOTA	
Oregon	9-0
Penn State	**8-1**
Indiana	8-1
Ohio State	7-2
Illinois	**6-3**
Iowa	**6-3**
Michigan	**5-4**
Minnesota	5-4
Rutgers	**4-5**
Washington	4-5
USC	**4-5**
UCLA	**3-6**
Nebraska	3-6
Michigan State	3-6
Wisconsin	**3-6**
Northwestern	2-7
Maryland	**1-8**
Purdue	0-9

Summary

- Conferences are too big to accurately determine who are the best teams in that conference
- The problem of imbalanced schedules is exacerbated when comparing teams from different conferences
- Many modern ranking systems (e.g., ESPN's Football Power Index) focus on *future potential* and are geared towards helping gamblers, not choosing the most deserving teams for a playoff
    - These systems often take into account recruiting rankings, previous years' results, and other biased metrics that favor some teams over others before any games are even played

Other potential applications

- Other sports
- Voting/elections
- Ranking board games/movies/musical artists
- Assigning grades in higher education

The Solution

Goals

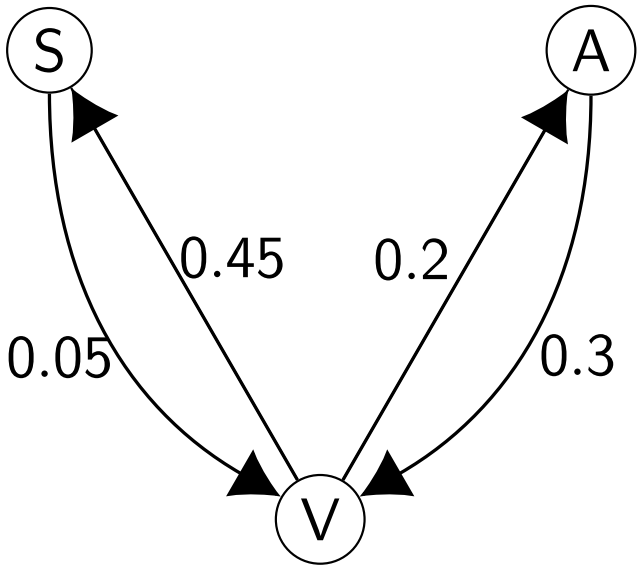
- Find a fairer way to determine the top teams in each conference
- Eliminate bias when comparing teams from different conferences
- Select the most *deserving* teams for the college football playoff
- Create an interesting project for my summer linear algebra class!

Main ideas

- All teams start the season with the same rating
- Teams earn rating points from every game they play
  - The amount earned is based on their opponent, the score of the game, and where it was played
- This creates a network of teams with rating points flowing through this network (technical term: signal-flow graph)
  - The final ratings for each team correspond to a "balanced" network or equilibrium point

A small example

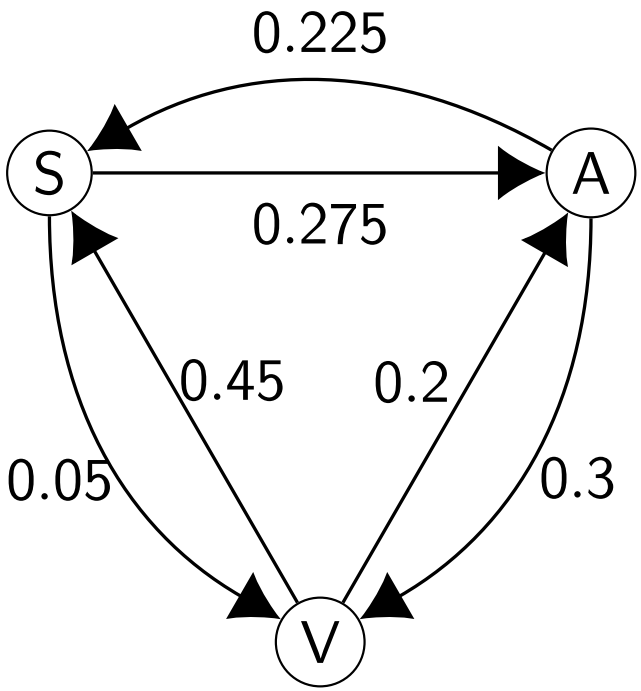
- Vanderbilt beats Alabama 40-35 at home
- South Carolina beats Vanderbilt 28-7 on the road



1.	South Carolina	1.66
2.	Vanderbilt	1.47
3.	Alabama	1.29

A small example, complicated

- Vanderbilt beats Alabama 40-35 at home
- South Carolina beats Vanderbilt 28-7 on the road
- Alabama beats South Carolina 27-25 at home



1. South Carolina 2.20
2. Alabama 1.94
3. Vanderbilt 1.69

Wait, where's the Linear Algebra??

- How do we find these equilibrium values for each team?
  1. Convert this graph into a system of equations
  2. Manipulate the system of equations into something solvable
  3. Use tools from linear algebra to find the solution

From graph to equations

$$
\begin{array}{ccccccccc}
p_{AA}x_A & + & p_{AS}x_S & + & p_{AV}x_V & + & 1 & = & x_A \\
p_{SA}x_A & + & p_{SS}x_S & + & p_{SV}x_V & + & 1 & = & x_S \\
p_{VA}x_A & + & p_{VS}x_S & + & p_{VV}x_V & + & 1 & = & x_V
\end{array}
$$

From graph to equations

$$
\begin{array}{ccccccccc}
(p_{AA}-1)x_A & + & p_{AS}x_S & + & p_{AV}x_V & + & 1 & = & 0 \\
p_{SA}x_A & + & (p_{SS}-1)x_S & + & p_{SV}x_V & + & 1 & = & 0 \\
p_{VA}x_A & + & p_{VS}x_S & + & (p_{VV}-1)x_V & + & 1 & = & 0
\end{array}
$$

From graph to equations

$$
\begin{array}{ccccccc}
(p_{AA}-1)x_A & + & p_{AS}x_S & + & p_{AV}x_V & = & -1 \\
p_{SA}x_A & + & (p_{SS}-1)x_S & + & p_{SV}x_V & = & -1 \\
p_{VA}x_A & + & p_{VS}x_S & + & (p_{VV}-1)x_V & = & -1
\end{array}
$$

From graph to equations

$$
\begin{array}{ccccccc}
-1x_A & + & p_{AS}x_S & + & p_{AV}x_V & = & -1 \\
p_{SA}x_A & + & -1x_S & + & p_{SV}x_V & = & -1 \\
p_{VA}x_A & + & p_{VS}x_S & + & -1x_V & = & -1
\end{array}
$$

From graph to equations

$$\begin{bmatrix} -1 & p_{AS} & p_{AV} \\ p_{SA} & -1 & p_{SV} \\ p_{VA} & p_{VS} & -1 \end{bmatrix} \begin{bmatrix} x_A \\ x_S \\ x_V \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

From graph to equations

$$\begin{bmatrix} x_A \\ x_S \\ x_V \end{bmatrix} = \begin{bmatrix} -1 & p_{AS} & p_{AV} \\ p_{SA} & -1 & p_{SV} \\ p_{VA} & p_{VS} & -1 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

The Schroeder Ranking

- We form a 135×135 matrix M with one row and column for each FBS team (plus one extra representing all non-FBS teams)
  - Label these teams $t_1, t_2, t_3, ..., t_{135}$
- For every team t_i, let $M_{i,i} = -1$
- For teams t_i and t_j that did not play each other, let $M_{i,j} = M_{j,i} = 0$
- For every game where t_i beat t_j, let $M_{i,j} = p$ and $M_{j,i} = q$, where $0 \leq q < p \leq \dfrac{1}{\#\text{ games played}}$
- **Key question:** How to determine p and q?

Results: Modified Big 10 standings

1.	Oregon	9-0
2.	Penn State	8-1
3.	Indiana	8-1
4.	Ohio State	7-2
5.	Illinois	6-3
6.	Iowa	6-3
7.	Michigan	5-4
8.	Minnesota	5-4
9.	Rutgers	4-5
10.	Washington	4-5
11.	USC	4-5
12.	UCLA	3-6
13.	Nebraska	3-6
14.	Michigan State	3-6
15.	Wisconsin	3-6
16.	Northwestern	2-7
17.	Maryland	1-8
18.	Purdue	0-9

1.	Oregon	2.450
2.	Ohio State	2.381
3.	Penn State	2.321
4.	Indiana	2.264
5.	Iowa	2.063
6.	Minnesota	1.998
7.	Michigan	1.965
8.	Illinois	1.944
9.	USC	1.942
10.	Rutgers	1.751
11.	Washington	1.750
12.	Nebraska	1.731
13.	Wisconsin	1.688
14.	UCLA	1.679
15.	Michigan State	1.500
16.	Northwestern	1.417
17.	Maryland	1.346
18.	Purdue	1.181

SEC Bias in CFP?

- The media: "Alabama deserves to be in the playoff!"
- Joel Klatt: "Tennessee's resume might be better than Notre Dame's and Penn State's, but it received a lower seed and has a tougher path to winning the title than those two teams."

Results: Modified CFP rankings

1.	**Oregon**
2.	**Texas**
3.	**Penn State**
4.	**Notre Dame**
5.	**Georgia**
6.	**Ohio State**
7.	**Tennessee**
8.	**SMU**
9.	**Indiana**
10.	**Boise State**
11.	Alabama
12.	Miami
13.	Ole Miss
14.	South Carolina
15.	**Arizona State**
16.	Iowa State
17.	**Clemson**
18.	BYU

1.	**Notre Dame**	2.637
2.	**Oregon**	2.585
3.	**Texas**	2.563
4.	**Ohio State**	2.543
5.	**SMU**	2.511
6.	**Boise State**	2.462
7.	**Penn State**	2.434
8.	Miami	2.411
9.	Alabama	2.404
10.	**Indiana**	2.400
11.	Ole Miss	2.398
12.	**Georgia**	2.388
13.	Iowa State	2.352
14.	BYU	2.345
15.	South Carolina	2.341
16.	**Arizona State**	2.311
17.	Louisville	2.290
18.	**Tennessee**	2.287
26.	**Clemson**	2.227

SEC Bias?

- Joel Klatt: "Tennessee's resume might be better than Notre Dame's and Penn State's, but it received a lower seed and has a tougher path to winning the title than those two teams."
- Really, Joel??

	Tennessee	**Notre Dame**	**Penn State**
Best win	9. Alabama	17. Louisville	35. Illinois
Second-best win	57. Florida	23. Army	41. Minnesota
Record vs Top 56	1-1	7-1	3-1
Games vs Bottom 10%	4	2	2

Going further

- Is there a better way to assign weights to each game?
- Can we use this ranking to settle debates from previous years?
- What happens if you apply this system to conferences as a whole?
- Does this method work in other settings (e.g., European football)?